Docket No. CA920000040US1 (7161-193U) **PATENT**

IN THE UNITED STATES PATENT AND TRADEMARK OFFICE
BEFORE THE BOARD OF PATENT APPEALS AND INTERFERENCES

In re Application of	:	Customer Number: 46320
Robert ENENKEL, et al.	:	Confirmation Number: 5643
Application No.: 10/008,473	:	Group Art Unit: 2123
Filed: November 9, 2001	:	Examiner: T. Stevens

For: METHOD AND APPARATUS FOR EVALUATING POLYNOMIALS AND RATIONAL FUNCTIONS

APPEAL BRIEF

Mail Stop Appeal Brief - Patents
Commissioner for Patents
P.O. Box 1450
Alexandria, VA 22313-1450

Sir:

This Appeal Brief is submitted in support of the Notice of Appeal filed April 10, 2006, and in response to the Notice of Non-Compliant Appeal Brief dated October 24, 2006, wherein Appellants appeal from the Examiner's rejection of claims 1-19 and 23-44.

I. REAL PARTY IN INTEREST

This application is assigned to IBM Corporation by assignment recorded on November 9, 2001, at Reel 012370, Frame 0073.

II. RELATED APPEALS AND INTERFERENCES

Appellants are unaware of any related appeals and interferences.

III. STATUS OF CLAIMS

Claims 1-19 and 23-44 are pending in this Application and have been three-times rejected. Claims 20-22 have been cancelled. It is from the multiple rejection of claims 1-19 and 23-44 that this Appeal is taken.

IV. STATUS OF AMENDMENTS

The claims have not been amended subsequent to the imposition of the Office Action dated January 25, 2006.

V. SUMMARY OF CLAIMED SUBJECT MATTER

Independent claims 1 and 23 are respectively directed to a machine-processing method and a machine for computing a property of a mathematically modeled physical system. The independent claims detail specific steps that are performed on a machine to output the value of a first polynomial as a floating point number. Independent claims 1 and 23 initially recite "reading, via a machine processing unit, input data including a value for each identified ordered coefficient of a first polynomial p(x) representing said property, said polynomial p(x) being expressed as $p(x) = \Sigma(Pj \cdot x^j)$ where j=0 to n, a value of a quantity x, a value of a predetermined x_i, and a value of a predetermined $p(x_i)$ correspondingly paired with said predetermined x_i," and support for this limitation is found in found Fig. 1, step 15, and also on pages 12 and 13 of Appellants' disclosure.

Claims 1 and 23 further recite "building, via said machine processing unit, a value of a second polynomial c(x) having ordered coefficients, said second polynomial c(x) being expressible as: $c(x) = \Sigma(C_k \cdot x^k)$ where k=0 to (n-1) so that said first polynomial p(x) is

expressible as: $p(x)=p(x_i)+\{x- x_i\} \bullet c(x)$," and support for this limitation is found in found Fig. 1, steps 30-50, and also on pages 14 and 15 of Appellants' disclosure with regard to equations 5, 5a, and 6. The building step further includes the steps of "i) determining, via said machine processing unit, a value for each ordered coefficient of said second polynomial c(x) by generating a plurality of machine processing unit signals to determine each said ordered coefficient of said second polynomial c(x) from: $C_k = \Sigma(P_{(k+1+j)} \bullet x^j_i)$ where j=0 to (n-1-k)" and "determining, via said machine processing unit, a value of said second polynomial c(x) by generating a plurality of machine processing unit signals to determine: $c(x) = \Sigma(C_k \bullet x^k)$ where k=0 to (n-1)" and support for these limitations are respectively found in Fig. 1, step 50, and also on pages 14 and 15 of Appellants' disclosure with regard to equation 6, and in Fig. 1, step 50, and also on pages 14 of Appellants' disclosure with regard to equation 5.

Claims 1 and 23 additionally recite "constructing, via said machine processing unit, a value of said first polynomial p(x) by generating a plurality of machine processing unit signals to determine: $p(x)=p(x_i)+\{x-x_i\} \bullet c(x)$," and support for this limitation is found in found Fig. 1, step 60, and also on page 14 of Appellants' disclosure with regard to equation 5a. Claims 1 and 23 finally recite "said value of the first polynomial is outputted as a floating point number and the floating point number is a digital representation of an arbitrary real number in said machine processing unit," which finds support on pages 1, 3, and 16 of Appellants' disclosure.

VI. GROUNDS OF REJECTION TO BE REVIEWED ON APPEAL

1. Claims 2, 24, and 44 were rejected under the second paragraph of 35 U.S.C. § 112; and

2. Claims 1-19 and 23-44 were rejected under 35 U.S.C. § 101.

VII. ARGUMENT

THE REJECTION OF CLAIMS 2, 24, AND 44 UNDER THE SECOND PARAGRAPH OF 35 U.S.C. § 112

For convenience of the Honorable Board in addressing the rejections, claim 24 stands or falls together with dependent claim 2, and claim 44 stands or falls alone.

On page 2 of the Office Action dated January 25, 2006 (hereinafter the third Office Action), the Examiner newly rejected originally-presented claims 2, 24, and 44. Specifically, the Examiner asserted that "[t]he term 'desired accuracy' in claims 2 and 24 is a relative term which renders the claim indefinite."

Appellants note that the term "desired accuracy" is used in the phrase "a difference between x and x_i is sufficiently small to achieve a desired accuracy of a final computation of said machine representation of a numerical value of said first polynomial p(x)." The term "desired accuracy" connotes that the accuracy of the final computation is not pre-defined and/or unchangeable, but instead, the accuracy of the final computation can be selected. Thus, one having ordinary skill in the art would be reasonably apprised of the scope of the term "desired accuracy" so as to distinguish accuracies that can be selected (i.e., desired) from accuracies that are pre-defined and/or unchangeable.

Claim 44 recites that "said library is operatively associated with a software programming language," and the Examiner has asserted that the "term 'operatively associated' in claim 44 is unclear." Claim 43, upon which claim 44 depends, recites "a computer-readable mathematical

software routine library including computer executable instructions for instructing a computer." Upon reading both these claims in concert, claims 43 and 44 recite a library that includes computer executable instructions that are "operatively associated" with a software programming language." Despite the Examiner's assertion to the contrary, Appellants are uncertain as to why the Examiner believes a claim reciting that computer instructions being associated with a software programming language is unclear. The Examiner has only concluded that the language is unclear without establishing (a) an interpretation of the claim in light of the specification; (b) an interpretation of the claim as interpreted by one of ordinary skill in the art; and (c) that the limitation(s) in the claim does not reasonably define the invention.[1]

Appellants position, therefore, is that one having ordinary skill in the art would have no difficulty understanding the scope of claims 2, 24, and 44, particularly when reasonably interpreted in light of the written description of the specification.

THE REJECTION OF CLAIMS 1-19 AND 23-44 UNDER 35 U.S.C. § 101

For convenience of the Honorable Board in addressing the rejections, claims 2-19 stand or fall together with independent claim 1, and claims 24-43 stand or fall together with independent claim 23.

Referring to pages 1 and 2 of Appellants' disclosure, the solving of polynomials by a computer is not perfect. The steps for computing binary representations of numbers can create an unacceptably large deviation between a computed binary representation and its theoretical numerical value due to successive rounding errors. Therefore, the "real" result from solving the

[1] See M.P.E.P. § 2173.02.

polynomial (if the polynomial is solvable) may vary from the computed result obtained by the computer, sometimes to a great degree. Although this information was informally provided to the Examiner, one having ordinary skill in the art would be aware of many instances where these differences between actual and computer-generated results from solving a polynomial have caused catastrophic failures. It is also well known that "rounding errors" and floating-point (i.e., a way of representing real numbers within a computer) are interrelated.

Referring to page 16 of Appellants' disclosure, it is well known that the precision of the floating-point number system in a particular computing system is definite and known, and this precision effects the rounding errors that are introduced into the process used by the computing system to solve the polynomial. Using the methodology described in claim 1, the present invention is directed to solving the polynomial with a greater precision than the inherent precision of the floating-point number system of the computing system. Therefore, the claimed invention is not just directed to solving an equation (i.e., the polynomial), as asserted by the Examiner. Instead, the claimed invention is directed to improving the precision by which a particular computer system (i.e., the performance) is able to solve a polynomial equation beyond the precision inherent provided by the floating-point number system of the computer system.

The Examiner arguments with regard to the rejection of claims 1-19 and 23-44 as being directed to non-statutory subject matter is found on pages 3 and 4 of the third Office Action. The Examiner, for example, has asserted that:

> In practical terms, claims define nonstatutory processes if they:
> - consist solely of mathematical operations without some claimed practical application (i.e., executing a "mathematical algorithm"); or
> - simply manipulate abstract ideas, e.g., a bid [Schrader, 22 F.3d at 293-94, 30 USPQ2d at 1458-59) or a bubble hierarchy (Warmerdam, 33 F.3d at 1360, 31 USPQ2d at 1759) without

> some claimed practical application.
> Thus, a claim that recites a computer that solely calculates a mathematical formula (see Benson) or a computer disk that solely stores a mathematical formula is not directed to the type of subject matter eligible for patent protection.

In re Shrader improperly relied upon

Notwithstanding the Examiner's reliance upon In re Shrader, the Federal Circuit recognizes that the court in In re Shrader did not apply a proper analysis. As stated in M.P.E.P. § 2106(I), "[o]ffice personnel should no longer rely on the Freeman-Walter-Abele test to determine whether a claimed invention is directed to statutory subject matter." The court in In re Shrader, however, relied on the Freeman-Walter-Abele test. The failure of the court in In re Shrader to rely upon a proper standard is discussed in AT&T Corp. v. Excel Communications, Inc.,[2] which states:

> Similarly, the court in In re Schrader relied upon the Freeman - Walter - Abele test for its analysis of the method claim involved. The court found neither a physical transformation nor any physical step in the claimed process aside from the entering of data into a record. See 22 F.3d at 294, 30 USPQ2d at 1458. The Schrader court likened the data-recording step to that of data-gathering and held that the claim was properly rejected as failing to define patentable subject matter. See id. at 294, 296, 30 USPQ2d at 1458-59. The focus of the court in Schrader was not on whether the mathematical algorithm was applied in a practical manner since it ended its inquiry before looking to see if a useful, concrete, tangible result ensued. Thus, in light of our recent understanding of the issue, the Schrader court's analysis is as unhelpful as that of In re Grams.

Therefore, the Examiner cannot properly rely upon In re Schrader to support the rejection of claims 1-19 and 23-44 under 35 U.S.C. § 101.

The Federal Circuit in AT&T Corp. also addressed In re Warmerdam[3], in which the Court stated:

> Finally, the decision in In re Warmerdam, 33 F.3d 1354, 31 USPQ2d 1754 (Fed. Cir. 1994) is not to the contrary. There the court recognized the difficulty in knowing exactly what a mathematical algorithm is, "which makes rather dicey the determination of whether the claim as a whole is no more than that." Id. at 1359, 31 USPQ2d at 1758. Warmerdam's claims 1-4 encompassed a method for controlling the motion of objects and machines to avoid collision with

[2] 172 F.3d 1352, 50 USPQ2d 1447 (Fed. Cir. 1999).
[3] 33 F.3d 1354, 31 USPQ2d 1754 (Fed Cir. 1994)

> other moving or fixed objects by generating bubble hierarchies through the use of a particular mathematical procedure. See id. at 1356, 31 USPQ2d at 1755-56. The court found that the claimed process did nothing more than manipulate basic mathematical constructs and concluded that "taking several abstract ideas and manipulating them together adds nothing to the basic equation"; hence, the court held that the claims were properly rejected under 101. Id. at 1360, 31 USPQ2d at 1759. Whether one agrees with the court's conclusion on the facts, the holding of the case is a straightforward application of the basic principle that mere laws of nature, natural phenomena, and abstract ideas are not within the categories of inventions or discoveries that may be patented under 101.

An algorithm is patentable when applied in "useful" way

In State Street Bank and Trust Company v. Signature Financial Group, Inc.,[4] the court elaborated on the mathematical algorithm exception to patentable subject matter by stating:

> Unpatentable mathematical algorithms are identifiable by showing they are merely abstract ideas constituting disembodied concepts or truths that are not "useful." From a practical standpoint, this means that to be patentable an algorithm must be applied in a "useful" way.

The court in State Street then set forth the criteria for establishing statutory subject matter under 35 U.S.C. § 101 as follows:

> The question of whether a claim encompasses statutory subject matter should not focus on which of the four categories of subject matter a claim is directed to —process, machine, manufacture, or composition of matter—but rather on the essential characteristics of the subject matter, in particular, its practical utility. Section 101 specifies that statutory subject matter must also satisfy the other "conditions and requirements" of Title 35, including novelty, nonobviousness, and adequacy of disclosure and notice. See In re Warmerdam, 33 F.3d 1354, 1359, 31 USPQ2d 1754, 1757-58 (Fed. Cir. 1994). For purpose of our analysis, as noted above, claim 1 is directed to a machine programmed with the Hub and Spoke software and admittedly produces a "useful, concrete, and tangible result." Alappat, 33 F.3d at 1544, 31 USPQ2d at 1557. This renders it statutory subject matter, even if the useful result is expressed in numbers, such as price, profit, percentage, cost, or loss.

Thus, as articulated above, the test for determining whether subject matter is patentable under 35 U.S.C. § 101 involves deciding if the subject matter produces a "useful, concrete, and tangible result." Furthermore, the law states that this result can be "expressed in numbers."

[4] 149 F.3d 1368, 47 USPQ2d 1596 (Fed Cir. 1998).

Appellants have established utility

A discussion of the procedural considerations regarding a rejection based upon lack of utility (i.e., 35 U.S.C. § 101) is found in M.P.E.P. § 2107.02. Specifically, M.P.E.P. § 2107.02(I) states that:

> regardless of the category of invention that is claimed (e.g., product or process), an applicant need only make one credible assertion of specific utility for the claimed invention to satisfy 35 U.S.C. 101 and 35 U.S.C. 112

In the paragraph spanning pages 1 and 2 of the disclosure and within the "Background of the Present Invention" section, Appellants stated the following:

> Steps for computing binary representations of numbers can create an unacceptably large deviation between an computer binary representation and its theoretical numerical value due to successive rounding errors. This can be an intolerable situation when a higher degree of accuracy is required. Various methods can improve computation accuracy but they may require a significant increase in processing time and/or hardware.

As recognized by those skilled in the art, a floating-point number is a digital representation of an arbitrary real number in a computer. As alluded to by Appellants in the above-reproduced passage, rounding errors[5] with floating point numbers can degrade computation accuracy. In the second full paragraph on page 16 of the disclosure, Appellants stated with regard to the invention the following:

> Optionally, accuracy can be further improved by choosing x_i such that $p(x_i)$ has extra accuracy beyond the precision of the floating-point number system of the computer.

Appellants, therefore, have asserted a credible utility (i.e., improving the precision of a floating-point number system in a computer).

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[5] E.g., 2/3 is not perfectly represented by .66666667.

As noted in M.P.E.P. § 2107.02(III)(A), the Court of Customs and Patent Appeals in In re Langer[6] stated the following:

> As a matter of Patent Office practice, a specification which contains a disclosure of utility which corresponds in scope to the subject matter sought to be patented must be taken as sufficient to satisfy the utility requirement of § 101 for the entire claimed subject matter unless there is a reason for one skilled in the art to question the objective truth of the statement of utility or its scope. (emphasis in original)

Since a credible utility is contained in Appellants' specification, the utility requirement of 35 U.S.C. § 101 (i.e., whether the invention produces a useful, concrete, and tangible result) has been met.

Claim 23

Although claims 1-19 are directed to a method, Appellants note that claims 23-44 are directed to a machine, and on this basis, without the need for further argument, claims 23-44 are directed to statutory subject matter. In the decision of In re Warmerdam,[7] the Court concluded that the method claims recited in claims 1-4 "[involve] no more than the manipulation of basic mathematical constructs, the paradigmatic 'abstract idea.'" The Court then sustained the rejection of claims 1-4 under 35 U.S.C. § 101.

However, the Court noted that claim 5 of the same application recited "[a] machine having a memory which contains data representing a bubble hierarchy generated by the method of any of Claims 1 through 4." With regard to this claim, the Court stated that "[c]laim 5 is for a machine, and is clearly patentable subject matter" despite the prior finding that method being performed by the machine was non-statutory subject matter. Therefore, since claims 23-44 are

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[6] 503 F.2d 1380, USPQ 288 (CCPA 1974).
7 3 F.3d 1354, 31 USPQ2d 1754 (Fed. Cir. 1994).

directed to a machine, then claims 23-44 are directed to statutory subject matter within the meaning of 35 U.S.C. § 101.

Conclusion

Based upon the foregoing, Appellants respectfully submit that the Examiner's rejections under 35 U.S.C. §§ 101, 112 are not viable. Appellants, therefore, respectfully solicit the Honorable Board to reverse the Examiner's rejections under 35 U.S.C. §§ 101, 112.

To the extent necessary, a petition for an extension of time under 37 C.F.R. § 1.136 is hereby made. Please charge any shortage in fees due under 37 C.F.R. §§ 1.17, 41.20, and in connection with the filing of this paper, including extension of time fees, to Deposit Account 09-0461, and please credit any excess fees to such deposit account.

Date: November 3, 2006

Respectfully submitted,

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VIII. CLAIMS APPENDIX

1. A machine-processing method for computing a property of a mathematically modelled physical system, the steps comprising:

a) reading, via a machine processing unit, input data including a value for each identified ordered coefficient of a first polynomial p(x) representing said property, said polynomial p(x) being expressed as $p(x) = \Sigma(Pj \bullet x^j)$ where j=0 to n, a value of a quantity x, a value of a predetermined x_i, and a value of a predetermined $p(x_i)$ correspondingly paired with said predetermined x_i;

b) building, via said machine processing unit, a value of a second polynomial c(x) having ordered coefficients, said second polynomial c(x) being expressible as: $c(x) = \Sigma(C_k \bullet x^k)$ where k=0 to (n-1) so that said first polynomial p(x) is expressible as: $p(x)=p(x_i)+\{x- x_i \} \bullet c(x)$, comprising the steps of:

i) determining, via said machine processing unit, a value for each ordered coefficient of said second polynomial c(x) by generating a plurality of machine processing unit signals to determine each said ordered coefficient of said second polynomial c(x) from: $C_k = \Sigma(P_{(k+1+j)} \bullet x^j{}_i)$ where j=0 to (n-1-k);

ii) determining, via said machine processing unit, a value of said second polynomial c(x) by generating a plurality of machine processing unit signals to determine: $c(x) = \Sigma(C_k \bullet x^k)$ where k=0 to (n-1);

c) constructing, via said machine processing unit, a value of said first polynomial p(x) by generating a plurality of machine processing unit signals to determine: $p(x)=p(x_i)+\{x-x_i\} \bullet c(x)$; and

d) outputting, via said machine-processing unit, said value of the first polynomial p(x) representing said property of the mathematically modelled physical system, wherein

said value of the first polynomial is outputted as a floating point number and the floating point number is a digital representation of an arbitrary real number in said machine processing unit.

2. The machine-implementable method of claim 1, wherein a difference between x and x_i is sufficiently small to achieve a desired accuracy of a final computation of said machine representation of a numerical value of said first polynomial p(x).

3. The machine-implementable method of claim 2 wherein the step of reading said input data comprises reading, via said machine processing unit, said input data from a machine-readable medium.

4. The machine-implementable method of claim 3 wherein said ordered coefficients of said second polynomial c(x) are computed from a mathematical expression derivable from: $C_k = \Sigma(P_{(k+1+j)} \bullet x^j_i)$ where j=0 to (n-1-k).

5. The machine-implementable method of claim 4 wherein said mathematical expression is a mathematical recurrence expression.

6. The machine-implementable method of claim 5 wherein said mathematical recurrence expression is a forward mathematical recurrence expression.

7. The machine-implementable method of claim 5 wherein said mathematical recurrence expression is a backward mathematical recurrence expression.

8. The machine-implementable method of claim 7 further adapted to implement said backward mathematical recurrence expression by comprising further steps for:

e) equating, via said machine-processing unit, a value of a highest ordered coefficient of said second polynomial c(x) to a value of an identified highest ordered coefficient of said first polynomial p(x) by generating a plurality of machine processing unit signals to determine: $C_{n-1}=P_n$; and

f) computing, via a machine processing unit, a value for each lower ordered coefficient of said second polynomial c(x) by generating a plurality of machine processing unit signals to determine:C_k-1=$x_i \bullet C_k+P_k$ where k = (n-1) to 1.

9. The machine-implementable method of claim 8 wherein said predetermined x_i is selected from a set of predetermined values of x_i.

10. The machine-implementable method of claim 9 wherein said predetermined x_i is a closest member of said set to said identified x.

11. The machine-implementable method of claim 10 wherein said step of determining a value of said second polynomial c(x) is computed by using Homer's Rule.

12. The machine-implementable method of claim 11 for determining a value of a denominator polynomial $q(x)$ having identified ordered denominator coefficients, said denominator polynomial $q(x)$ being expressible as: $q(x) = \Sigma(Q_j \bullet x^j)$ where $j=0$ to m, comprising further steps of:

g) adapting said input data to further include a value for each identified ordered denominator coefficient of said denominator polynomial $q(x)$, a value of a predetermined $q(x_i)$ correspondingly paired with said predetermined x_i; and

h) determining, via said machine processing unit, a value of another polynomial $d(x)$ having ordered denominator coefficients, said another polynomial $d(x)$ being expressible as: $d(x) = \Sigma(D_k \bullet x^k)$ where $k = 0$ to $(m-1)$ so that said denominator polynomial $q(x)$ is expressible as: $q(x) = q(x_i)+\{x- x_i \} \bullet d(x)$, and a value for the said denominator polynomial is resolved.

13. The machine-implementable method of claim 12 wherein the first polynomial $p(x)$ is a numerator polynomial $p(x)$, and $p(x)-p(x_i)$ is computed, and $p(x_i)$ is not read.

14. The machine-implementable method of claim 13 for determining a value of a rational function $r(x)$ being expressible as a quotient of said numerator polynomial $p(x)$ and said denominator polynomial $q(x)$ expressed as $r(x) = p(x) / q(x)$, comprising further steps of:

i) adapting said input data to further including a value of a predetermined $r(x_i)$ correspondingly paired with said predetermined x_i; and

j) constructing, via said machine processing unit, a value of said rational function $r(x)$ by generating a plurality of machine processing unit signals to determine:

$r(x) = r(x_i) \bullet (1 - (q (x)-q (x_i))/q(x)) + (p(x) - p(x_i))/q(x)$.

15. The machine-implementable method of claim 14 wherein said rational function r(x) is an approximation to a Bessel function.

16. The machine-implementable method of claim 14 wherein said rational function r(x) is an approximation to an error function (ERF).

17. The machine-implementable method of claim 14 wherein said rational function r(x) is an approximation to a complementary error function (ERFC).

18. The machine-implementable method of claim 14 wherein said rational function r(x) is an approximation to a log gamma function (LGAMMA).

19. The machine-implementable method of claim 11 or 14 wherein all values are machine representations of some numerical value, said machine processing unit is a computer processing unit, each machine representation is a binary representation operable with said computer processing unit, and machine-readable medium is a computer-readable medium.

Claims 20-22 (Cancelled)

23. A machine for computing a property of a mathematically modelled physical system, the machine configured to perform the steps comprising:

a) reading, via a machine processing unit, input data including a value for each identified ordered coefficient of a first polynomial p(x) representing said property, said polynomial p(x) being expressed as $p(x) = \Sigma(P_j \bullet x^j)$ where j = 0 to n, a value of a quantity x, a value of a predetermined x_i, and a value of a predetermined $p(x_i)$ correspondingly paired with said predetermined x_i;

b) building, via said machine processing unit, a value of a second polynomial c(x) having ordered coefficients, said second polynomial c(x) being expressible as: $c(x) = \Sigma(C_k \bullet x^k)$ where k = 0 to (n-1) so that said first polynomial p(x) is expressible as: $p(x) = p(x_i)+\{x-x_i\} \bullet c(x)$, comprising the steps of:

i) determining, via said machine processing unit, a value for each ordered coefficient of said second polynomial c(x) by generating a plurality of machine processing unit signals to determine each said ordered coefficient of said second polynomial c(x) from: $C_k=\Sigma(P_{(k+1+j)}\ x^j_i)$ where j= 0 to (n-1-k);

ii) determining, via said machine processing unit, a value of said second polynomial c(x) by generating a plurality of machine processing unit signals to determine: $c(x) = \Sigma(C_k \bullet x^k)$ where k = 0 to (n-1);

c) constructing, via said machine processing unit, a value of said first polynomial p(x) by generating a plurality of machine processing unit signals to determine: $p(x)=p(x_i)+\{x- x_i\} \bullet c(x)$; and

d) outputting, via said machine-processing unit, said value of the first polynomial p(x) representing said property of the mathematically modelled physical system, wherein

said value of the first polynomial is outputted as a floating point number and the floating point number is a digital representation of an arbitrary real number in said machine for computing.

24. The machine of claim 23 wherein a difference between x and x_i is sufficiently small to achieve a desired accuracy of a final computation of said machine representation of a numerical value of said first polynomial p(x).

25. The machine of claim 24 wherein said means for reading said input data comprises means for reading, via said machine processing unit, said input data from a machine-readable medium.

26. The machine of claim 25 wherein said ordered coefficients of said second polynomial c(x) are computed from a mathematical expression derivable from: $C_k = \Sigma(P_{(k+1+j)} \bullet x^j_i)$ where j= 0 to (n-1-k).

27. The machine of claim 26 wherein said mathematical expression is a mathematical recurrence expression.

28. The machine of claim 27 wherein said mathematical recurrence expression is a forward mathematical recurrence expression.

29. The machine of claim 27 wherein said mathematical recurrence expression is a backward mathematical recurrence expression.

30. The machine of claim 29 further adapted to implement said backward mathematical recurrence expression by further comprising:

e) means for equating, via said machine processing unit, a value of a highest ordered coefficient of said second polynomial c(x) to a value of an identified highest ordered coefficient of said first polynomial p(x) by generating a plurality of machine processing unit signals to determine: $C_{n-1} = P_n$; and

f) means for computing, via said machine processing unit, a value for each lower ordered coefficient of said second polynomial c(x) by generating a plurality of machine processing unit signals to determine: $C_{k+1} = x_i \bullet C_k + P_k$ where k = (n-1) to 1.

31. The machine of claim 30 wherein said predetermined x_i is selected from a set of predetermined values of x_i.

32. The machine of claim 30 wherein said predetermined x_i is a closest member of said set to said identified x.

33. The machine of claim 32 wherein the determining means for determining a value of said second polynomial c(x) is computed by using Homer's Rule.

34. The machine of claim 33 for determining a value of a denominator polynomial $q(x)$ having identified ordered denominator coefficients, said denominator polynomial $q(x)$ being expressible as: $q(x) = \Sigma(Q_j \bullet x^j)$ where $j = 0$ to m, comprising further steps of:

g) adapting said input data to further include a value for each identified ordered denominator coefficient of said denominator polynomial $q(x)$, and a value of a predetermined $q(x_i)$ correspondingly paired with said predetermined x_i; and

h) determining, via said machine processing unit, a value of another polynomial $d(x)$ having ordered denominator coefficients, said another polynomial $d(x)$ being expressible as: $d(x) = \Sigma(D_k \bullet x^k)$ where $k = 0$ to $(m-1)$ so that said denominator polynomial $q(x)$ is expressible as: $q(x)=q(x_i)+\{x- x_i \} \bullet d(x)$, and a value for the said denominator polynomial is resolved.

35. The machine of claim 34 wherein the first polynomial $p(x)$ is a numerator polynomial $p(x)$, and $p(x)-p(x_i)$ is computed, and $p(x_i)$ is not read.

36. The machine of claim 35 for determining a value of a rational function $r(x)$ being expressible as a quotient of said numerator polynomial $p(x)$ and said denominator polynomial $q(x)$ expressed as $r(x) = p(x) / q(x)$, comprising further steps of:

i) adapting said input data to further including a value of a predetermined $r(x_i)$ correspondingly paired with said predetermined x_i; and

j) constructing, via said machine processing unit, a value of said rational function $r(x)$ by generating a plurality of machine processing unit signals to determine:

$$r(x) = r(x_i) \bullet (1 - (q (x)-q (x_i))/q(x))) + (p(x) - p(x_i))/q(x).$$

37. The machine of claim 36 wherein said rational function is an approximation to a Bessel function.

38. The machine of claim 36 wherein said rational function is an approximation to an error function (ERF).

39. The machine of claim 36 wherein said rational function is an approximation to a complementary error function (ERFC).

40. The machine of claim 36 wherein said rational function is an approximation to a log gamma function (LGAMMA).

41. The machine of claim 33 or 36 wherein all values are machine representations of some numerical value, said machine processing unit is a computer processing unit, each machine representation is a binary representation operable with said computer processing unit, and said machine-readable medium is a computer-readable medium.

42. A machine having a computer-readable program product having computer executable instructions for instructing a computer to embody the machine of claim 41.

43. A machine having a computer-readable mathematical software routine library including computer executable instructions for instructing a computer to embody the machine of claim 41.

44. A machine having the computer-readable mathematical software routine library of claim 43 wherein said library is operatively associated with a software programming language.

IX. EVIDENCE APPENDIX

No evidence submitted pursuant to 37 C.F.R. §§ 1.130, 1.131, or 1.132 of this title or of any other evidence entered by the Examiner has been relied upon by Appellants in this Appeal, and thus no evidence is attached hereto.

X. RELATED PROCEEDINGS APPENDIX

Since Appellants are unaware of any related appeals and interferences, no decision rendered by a court or the Board is attached hereto.